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Tuning Rules for Passivity-Preserving Controllers

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Abstract

Nonlinear Passivity-based control (PBC) algorithms for power converters have proven to be an interesting alternative for other, mostly linear, control techniques. The control objective is usually achieved through an energy reshaping process and by injecting damping to modify the dissipation structure of the system. However, a key question that arises during the implementation of the controller is how to tune the various parameters. From previous work we know that a PBC controller forces the closed-loop dynamics to behave as if there are artificial resistors connected to the real circuit elements. This has led to conservative tuning rules stemming from characteristic impedance matching conditions. In this paper an alternative solution is provided that uses the classical Brayton-Moser equations. The criteria derived from these equations result in fairly sharp and less conservative tuning rules to guarantee stability and non-oscillatory responses. Both criteria are compared and tested using the elementary single-switch boost converter.

1 Introduction

In recent years passivity-based control (PBC) design for switch-mode power converters has become quite an active area in both the field of system and control theory and power electronics. This technique stems from classical Euler-Lagrange dynamics and the closely related field of robotics. The application to single-switch DC-to-DC power converters was first proposed by Sira-Ramírez *et al.* [9] and is generalized to larger networks, like the coupled-inductor Ćuk converter, three-phase rectifiers and inverters, in e.g. [2, 5, 8] and the references therein. In these works it is shown that a PBC design method is applicable to the average pulse-width modulated (PWM) models of switch regulated power converters, provided that such (idealized) models correspond to systems derivable from the classical Euler-Lagrange (EL) dynamics theory. One of the major advantages of underscoring the physical structure, e.g., energy and interconnection, of these networks is that the nonlinear phenomena and features are explicitly incorporated in the model, and thus in the corresponding PBC. This in contrast to conventional techniques which are mainly based on linearized dynamics and corresponding PI, PID or lead-lag control. Many power electronic

converters are nonlinear non-minimum phase systems where the resonance frequency is varying with the desired output voltages or currents. For that reason, controllers stemming from linear techniques are sometimes difficult to tune as to ensure robust performance, especially in the presence of large setpoint changes and disturbances that cause circuit operation to deviate from the nominal point of operation.

The basic idea behind PBC design is to modify the energy of the system and add damping by modification of the dissipation structure. During this process two fundamental questions arise: "*Which variable(s) have to be stabilized to a certain value in order to regulate the output(s) of interest toward a desired equilibrium value? In other words, are the zero-dynamics of the output(s) to be controlled stable with respect to the available control input(s), and if not, for which state variables is it stable?*", and "*Where to inject the damping and how to tune the various parameters associated to the energy modification and to the damping assignment stage?*". In general it remains hard to give an answer to the first question for general circuits since we are not able to give explicit formulations of the zero-dynamics for a general converter structure. Because the PBC relies on a partial system inversion we can not control the non-minimum phase states directly. Application of PBC to for example the boost, buck-boost [9] and the (coupled-inductor) Ćuk [8] converters leads to an indirect regulation scheme of the output voltage through regulation of the input current.

The first attempt to develop some guidelines to adjust the damping parameters is to study the disturbance attenuation properties and look for upper and lower bounds on these parameters using \mathcal{L}_2 -gain analysis techniques [7]. Unfortunately, the necessary calculations become quite complex and cumbersome, especially when dealing with large converter structures. In previous works the location where to add the damping is mainly motivated by the form of the dissipation structure, in the sense that damping is added to those states that do not contain any damping terms a priori. For example, in the boost converter case this means that only damping is injected on the input current (series damping), as the output voltage already contains a damping term due to the load resistance. However, this leads to a PBC regulated circuit that is highly sensitive to load variations and also needs an expensive current sensor to measure the inductor current. This disadvantage holds for many other switching networks too.

Recently, in [2] we have proposed a solution to overcome this problem using the concept of parallel damping. Additionally, this approach enables us to control a non-minimum phase system by measuring its non-minimum phase outputs only.

The main contribution of the work presented in this paper can be summarized as follows. In Section 2, we briefly introduce the classical Brayton-Moser equations (BM) and accommodate them for the inclusion of controlled switches. Interestingly enough, due to their passive nature, the BM equations also appear to be naturally suited for application of PBC. Because the PBC design is based on the energy and the interconnection structure of the circuit, it is not surprising that this allows an interpretation in similar physical terms of the controlled closed-loop system. From a circuit-theoretic point of view, as is discussed in [2], the controller produces a computed duty ratio function which forces the closed-loop dynamics to act as if there are *virtual* resistors connected in series and/or in parallel to the real circuit elements. This directly leads us to an alternative methodology to tune the various control parameters based on modified versions of the stability theorems proposed in [1]. The fairly sharp criteria following from these theorems offer a systematic and straight forward tool for solving the tuning problem. The details are worked out in Section 3. Section 4 treats a tuning example using the single-switch boost converter illustrating both the concept of series and parallel damping injection. Finally, we conclude the paper with some computer simulations and the conclusions.

2 The Switched Brayton-Moser Equations

In [3] we have shown that the classical Brayton-Moser equations [1] may be written as

$$\frac{d}{dt} \left[\frac{\partial H^*}{\partial x}(x) \right] = \Upsilon \frac{\partial P}{\partial x}(x). \quad (1)$$

Here $x = [x_1, \dots, x_\sigma, x_{\sigma+1}, \dots, x_n]^T \in \mathbb{R}^n$, $n = \sigma + \rho$, denote the currents through the σ inductors and the voltage across the ρ capacitors, and $\Upsilon = \text{diag}[I_{\sigma \times \sigma}, -I_{\rho \times \rho}]$. The total stored co-energy is defined by $H^*(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ as

$$H^*(x) = \sum_{j=1}^{\sigma} \int_0^{x_j} \mathcal{L}_j^*(x'_1, \dots, x'_\sigma) dx'_j + \sum_{k=\sigma+1}^n \int_0^{x_k} \mathcal{C}_k^*(x'_{\sigma+1}, \dots, x'_n) dx'_k,$$

where $\mathcal{L}_j^*(x_1, \dots, x_\sigma)$ and $\mathcal{C}_k^*(x_{\sigma+1}, \dots, x_n)$ denote the constitutive relations of the inductors and the capacitors, respectively. The scalar function $P(x)$ is called the mixed-potential function, to be specified later. As stated in [4], equations (1) do not establish a Lagrangian system in the classical sense, but they can be viewed as some *degenerate* Lagrangian form. On the other hand, in [3] it is shown that if $\mathcal{L}_j^*(x_1, \dots, x_\sigma)$ and $\mathcal{C}_k^*(x_{\sigma+1}, \dots, x_n)$ are invertible then

(1) establishes a *port-controlled co-Hamiltonian* (PCH*) system due to the fact that its structure is dual in the coordinate and energy sense to a PCH system [6]. One of our main motivations to use the Brayton-Moser equations is that the dynamics are directly expressed in terms of measurable quantities (currents and voltages), while Lagrangian or Hamiltonian dynamics are usually based on charges and/or fluxes.

The next step is to accommodate (1) to properly define the dynamic behavior of circuits with controllable switches. Without loss of generality we restrict ourselves to circuits containing voltage sources only. In that case the mixed-potential function can be decomposed into

$$P(x) = [P_T + (P_E - P_R) + P_G](x), \quad (2)$$

where $P_T(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ represents the internal power preserved at the ports of the circuit following from Kirchhoff's laws, to be specified in a moment. The scalars $P_R(x) : \mathbb{R}^\sigma \rightarrow \mathbb{R}$ and $P_G(x) : \mathbb{R}^\rho \rightarrow \mathbb{R}$ are related to the dissipated power in the resistive and conductive elements, respectively, i.e.,

$$P_R(x_1, \dots, x_\sigma) = \sum_{i=1}^{\sigma} \int_0^{x_i} \mathcal{R}_i(x'_1, \dots, x'_\sigma) dx'_i$$

$$P_G(x_{\sigma+1}, \dots, x_n) = \sum_{j=\sigma+1}^n \int_0^{x_j} \mathcal{G}_j(x'_{\sigma+1}, \dots, x'_n) dx'_j,$$

where $\mathcal{R}(x_1, \dots, x_\sigma)$ and $\mathcal{G}(x_{\sigma+1}, \dots, x_n)$ denote the constitutive relations of the resistances and the conductances (Ohm's law). In a similar fashion we define the total supplied power $P_E(x) : \mathbb{R}^\sigma \rightarrow \mathbb{R}$ as

$$P_E(x_1, \dots, x_\sigma) = \sum_{k=1}^{\sigma} \int_0^{x_k} \mathcal{E}_k(x'_1, \dots, x'_\sigma) dx'_k.$$

For circuits that contain one or more switches, we denote the switch position(s) by $u = [u_1, \dots, u_m]$, where $u_j \in U := \{0, 1\}$, $j = 1, \dots, m$, i.e., ON or OFF, or in other words u is in the discrete set U^m . Depending on the application, re-definition of the switching function may also result in for example $u_j \in U := \{-1, 0, 1\}$. The mixed-potential function is modified to include switching functions u by letting $P(x) = P^u(x)$. For circuits containing a single switch the separate potentials $P_\gamma^u(x)$ are defined as

$$P_\gamma^u(x) := u P_\gamma^1(x) + (1 - u) P_\gamma^0(x), \quad (3)$$

where $\gamma \in \{T, E, R, G\}$, $P_\gamma^1(x)$ are the potentials¹ for the switch position $u = 1$, and $P_\gamma^0(x)$ are the potentials for the switch position $u = 0$. The dynamics of a switched system are then expressed by means of a five-tuple Σ^u called the *BM-parameters*:

$$\Sigma^u = \{H^*(x), P_T^u(x), P_R^u(x), P_E^u(x), P_G^u(x)\}. \quad (4)$$

¹Notice that the way the switching function enters the potential function as defined in (3) differs from the definition of the switched Rayleigh dissipation function as defined in [2, 5, 8] in the sense that here we have used the concept of superposition of the power flows.

Consequently, for every admissible switch vector $u \in U$ we have a different but unique set of parameters Σ^u . At this point, it is worth mentioning that $P_T(x)$ reflects the power preserving interconnection (Dirac) structure, deduced from Kirchhoff's laws, as defined for the PCH systems (see [1, 3] for more details), i.e.,

$$\frac{\partial^2 P_T^u}{\partial x^2}(x) = - \left[\frac{\partial^2 P_T^u}{\partial x^2}(x) \right]^T, \text{ (skew-symm.)},$$

which forms a necessary and sufficient condition for the application of passivity-based controller design [5, 6].

Example 1 Let us consider the BM dynamics of the DC-to-DC boost type converter [5] with a linear inductance L , a linear capacitor C and a nonlinear unknown load conductance described by the characteristic function $\mathcal{G}(x_2)$. Furthermore let x_1 denote the current through the inductor, x_2 denote the voltage across the capacitance and let R represent the losses of the switch u and the source E . In terms of the BM-parameters the dynamics are completely determined by letting

$$\begin{aligned} P_T^u(x) &= -(1-u)x_1x_2 \\ P_R^u(x) &= \frac{1}{2}Rx_1^2 \\ P_G^u(x) &= \int_0^{x_2} \mathcal{G}(x_2')dx_2' \\ P_E^u(x) &= x_1E, \end{aligned}$$

and define the total co-energy stored in the circuit

$$H^*(x) = \int_0^{x_1} Lx_1'dx_1' + \int_0^{x_2} Cx_2'dx_2'.$$

Hence, after plugging the latter into (1), the differential equations describing the circuit are given by

$$\begin{aligned} L\dot{x}_1 &= E - Rx_1 - (1-u)x_2 \\ C\dot{x}_2 &= (1-u)x_1 - \mathcal{G}(x_2). \end{aligned} \quad (5)$$

The aim of defining the above properties is to reveal and to study the physical relations in terms of the five-tuple defined in (4). In the next section, these properties form the basis for the developments to arrive at a PBC design. For completeness, we note that the switched BM equations are also closely related to the average PWM (pulse-width modulation) models (See, e.g., [5] for a detailed discussion on this subject in the Lagrangian framework). This means that x is replaced by the average state z , representing the average inductor currents and capacitor voltages, and the discrete control u is replaced by its duty ratio function vector μ . For the average value of the switch position we thus have the following consistency conditions

$$\begin{aligned} P_\gamma^\mu(z)|_{\mu=1} &= P_\gamma^1(z) \\ P_\gamma^\mu(z)|_{\mu=0} &= P_\gamma^0(z). \end{aligned}$$

Note that $P_\gamma^u(z)$ can be considered as a weighted ratio, with weighting parameter μ , between $P_\gamma^1(z)$ and $P_\gamma^0(z)$. In the sequel we will use the average models with z as the states and μ as the duty ratio of the switch, operating in the closed set U .

3 PBC in the BM Framework

We now continue with the average PWM models. In the Lagrangian framework the design of a passivity-based controller (PBC) is to modify the Lagrangian, and add damping by modification of the Rayleigh dissipation function, see [5]. For the Brayton-Moser framework this means that we have to modify the co-energy function $H^*(z)$ and the dissipative potentials $P_R^\mu(z)$ and/or $P_G^\mu(z)$ of (4) to arrive at a desired co-energy and dissipation functions that preserve the original structure and ensure asymptotic stability. To do this, let $\tilde{z} := z - \xi$ define the error state, and let ξ denote the desired auxiliary states of the controller. Furthermore, for sake of brevity we split the mixed-potential into an interconnection part $P_T^\mu(x)$, a supplied part $P_E^\mu(x)$ and a dissipated part defined by $P_D^\mu(x)$, where $P_D^\mu(x) = P_R^\mu(x) + P_G^\mu(x)$. Notice that $P_D^\mu(x)$ can be considered as some sort of generalized Rayleigh dissipation function. Hence, we may define the desired closed-loop error co-energy and dissipation functions

$$H_M^*(z, \xi) = H^*(z)|_{z=\tilde{z}} \quad (6)$$

$$P_M^\mu(z, \xi) = P_D^\mu(z)|_{z=\tilde{z}} + P_V^\mu(z, \xi), \quad (7)$$

with $P_V^\mu(z, \xi)$ the injected dissipation (damping) and where we have assumed that the voltage sources do not explicitly depend on the inductor currents. Consequently, the minimum of these functions will be located at the desired equilibrium points $\tilde{z} = \xi$. The average closed-loop error dynamics associated to the desired storage function (6) are then given by

$$\frac{d}{dt} \left[\frac{\partial H_M^*}{\partial \tilde{z}}(\tilde{z}) \right] - \Upsilon \frac{\partial P_T^\mu}{\partial \tilde{z}}(\tilde{z}) + \frac{\partial P_M^\mu}{\partial \tilde{z}}(\tilde{z}) = \Phi, \quad (8)$$

where the term Φ is defined as

$$\begin{aligned} \Phi := & \frac{\partial P_E^\mu}{\partial \xi}(\xi) - \frac{d}{dt} \left[\frac{\partial H_M^*}{\partial \xi}(\xi) \right] + \Upsilon \frac{\partial P_T^\mu}{\partial \xi}(\xi) \\ & - \frac{\partial P_D^\mu}{\partial \xi}(\xi) + \frac{\partial P_V^\mu}{\partial \tilde{z}}(\tilde{z}). \end{aligned} \quad (9)$$

The next step of the design procedure is to derive, using (9), the control signals μ and the relations for ξ , required to assign the desired storage function and thus to ensure that $\Phi \equiv 0$. That is, to achieve the co-energy reshaping plus damping injection. Hence, the closed-loop error dynamics satisfy

$$\frac{d}{dt} \left[\frac{\partial H_M^*}{\partial \tilde{z}}(\tilde{z}) \right] - \Upsilon \frac{\partial P_T^\mu}{\partial \tilde{z}}(\tilde{z}) + \frac{\partial P_M^\mu}{\partial \tilde{z}}(\tilde{z}) = 0. \quad (10)$$

The implicit definition of the control law which ensures the closed-loop error dynamics to be of the form (10) is obtained by

$$\begin{aligned} \frac{d}{dt} \left[\frac{\partial H_M^*}{\partial \xi}(\xi) \right] = & \Upsilon \frac{\partial P_T^\mu}{\partial \xi}(\xi) - \frac{\partial P_D^\mu}{\partial \xi}(\xi) \\ & + \frac{\partial P_E^\mu}{\partial \xi}(\xi) + \frac{\partial P_V^\mu}{\partial \tilde{z}}(\tilde{z}). \end{aligned} \quad (11)$$

Finally, an explicit definition of the control law is obtained after solving for μ with respect to a minimum phase state (or

states) as discussed in the introduction. This results for the controller in a partial system inversion. It is well-known from Lyapunov's stability theory that if $H_M^*(\tilde{z})$ along the solution of (10) strictly decreases with time except at the equilibrium points, then an equilibrium solution is asymptotically stable if and only if $H_M^*(\tilde{z})$ has, at least, a local minimum there. It is easily checked that for every admissible $Q_M^\mu(\tilde{z}) > 0$

$$\dot{H}_M^*(\tilde{z}) = -Q_M^\mu(\tilde{z}) < 0, \forall \tilde{z} \neq 0, \quad (12)$$

with $Q_M^\mu(\tilde{z})$ the total dissipated power. Notice that in case $P_M^\mu(\tilde{z})$ is polynomial then $Q_M^\mu(\tilde{z}) = \alpha P_M^\mu(\tilde{z})$, with α some positive constant (in the linear case $\alpha = 2$). From (12) we may conclude that, if the error dynamics coincide with (10), the closed-loop error behavior \tilde{z} is asymptotically stable at zero, i.e., $\tilde{z} \rightarrow 0$ as $t \rightarrow \infty$.

So far we have derived the procedure to obtain a PBC strategy from the BM equations, as is developed in [5] for the Lagrangian framework. However, since (10) can be considered as a system resulting from standard feedback interconnection, the closed-loop system forms a BM or a passivity-preserving interconnection. For that, we are able to use some interesting and important ideas developed in [1]. Interesting enough, in [1], Theorem 3 and 4, page 19 and 21, stability criteria are developed that use the mixed-potential function. These criteria can be used to rule out the existence of self-sustained oscillations. Moreover, if we translate their ideas to the setting as presented above, we have strong criteria to tune the various control parameters. In other words, we can assign values to the injected dissipation functions to assure a desired dynamic behavior in terms of, for example, overshoot and robustness against load variations. Brayton and Moser's theorems can be restated and accommodated for the inclusion of the functions μ as follows. For the closed-loop error mixed-potential function $P^\mu(\tilde{z})$, a Lyapunov-based stability condition for the system (10) is stated as follows. Let $\tilde{z} = [\tilde{v}^T, \tilde{w}^T]^T$, where $\tilde{v} = [\tilde{z}_1, \dots, \tilde{z}_\sigma]^T$ and $\tilde{w} = [\tilde{z}_{\sigma+1}, \dots, \tilde{z}_\rho]^T$, denote the error-currents through the inductors and error-voltages across the capacitors. Furthermore, let

$$\begin{aligned} \mathcal{L}(\tilde{v}) &= \frac{\partial^2 H^*}{\partial \tilde{v}^2}(\tilde{z}), \quad \mathcal{C}(\tilde{w}) = \frac{\partial^2 H^*}{\partial \tilde{w}^2}(\tilde{z}), \\ \mathcal{R}(\tilde{v}) &= \frac{\partial^2 P_M^\mu}{\partial \tilde{v}^2}(\tilde{z}), \quad \mathcal{G}(\tilde{w}) = \frac{\partial^2 P_M^\mu}{\partial \tilde{w}^2}(\tilde{z}) \end{aligned}$$

represent the inductance matrix, the capacitance matrix, the resistance matrix and the conductance matrix, respectively, and let

$$\Upsilon \frac{\partial^2 P_T^\mu}{\partial \tilde{z}^2}(\tilde{z}) = \begin{bmatrix} 0_{\sigma \times \sigma} & -\psi(\mu) \\ \psi^T(\mu) & 0_{\rho \times \rho} \end{bmatrix},$$

where ψ is a matrix of appropriate dimensions, possibly depending on the switching functions, deduced from the Kirchhoff's laws, then

Theorem 1 *If \mathcal{R} is a positive definite constant matrix, $\int_0^{\tilde{w}} \mathcal{G}(\tilde{w}') d\tilde{w}' + |-\psi(\mu)\tilde{w}| \rightarrow 0$ as $|\tilde{w}| \rightarrow 0$, and*

$$\|K_R\| := \left\| \mathcal{L}^{\frac{1}{2}}(\tilde{v}) \mathcal{R}^{-1}(-\psi(\mu)) \mathcal{C}^{-\frac{1}{2}}(\tilde{w}) \right\| < 1,$$

with $0 \leq \mu < 1$, then for all (\tilde{v}, \tilde{w}) the solutions of (10) tend (in a non-oscillatory way) to the set of equilibrium points $(\tilde{v}, \tilde{w}) \rightarrow 0$ as $t \rightarrow \infty$.²

Although it is assumed that \mathcal{R} is constant in the first place, the criterion of Theorem 1 places a constraint on \mathcal{R} that bounds the lower limit in terms of the possibly nonlinear storage elements, $\mathcal{L}(\tilde{v})$ and $\mathcal{C}(\tilde{w})$. Therefore, it may occur that \mathcal{R} needs to be chosen as a function of \tilde{v} and \tilde{w} in order to fulfill (13), i.e., $\mathcal{R}(\tilde{v}, \tilde{w})$, and therefore may become nonlinear as well. Notice that if Theorem 1 is satisfied, stability is guaranteed regardless of $\mathcal{G}(\tilde{w})$! A similar criterion for the \mathcal{G} -matrix is derived as follows

Theorem 2 *If \mathcal{G} is a positive definite constant matrix, $\int_0^{\tilde{v}} \mathcal{R}(\tilde{v}') d\tilde{v}' + |\psi^T(\mu)\tilde{v}| \rightarrow 0$ as $|\tilde{v}| \rightarrow 0$, and*

$$\|K_G\| := \left\| \mathcal{C}^{\frac{1}{2}}(\tilde{w}) \mathcal{G}^{-1} \psi^T(\mu) \mathcal{L}^{-\frac{1}{2}}(\tilde{v}) \right\| < 1,$$

with $0 \leq \mu < 1$, then for all (\tilde{v}, \tilde{w}) the solutions of (10) tend (in a non-oscillatory way) to the set of equilibrium points $(\tilde{v}, \tilde{w}) \rightarrow 0$ as $t \rightarrow \infty$.

Detailed proofs for the case that ψ is a constant matrix are given in [1]. The proofs for $\psi(\mu)$, for all $0 \leq \mu < 1$, follow in a similar way. Notice that the criteria of Theorem 1 and Theorem 2 enables us to choose between two different damping injection strategies. Theorem 1 suggest to add damping on all the inductor currents by injecting series resistances, while the criterion of Theorem 2 suggests to inject damping on the capacitor voltages by injecting parallel conductances, i.e., we should modify either $P_R^\mu(z)$ or $P_G^\mu(z)$ by letting

$$P_M^\mu(\tilde{z}) = P_D^\mu(z)|_{z=\tilde{z}} + P_V^\mu(\tilde{v}), \quad \text{or} \quad (13)$$

$$P_M^\mu(\tilde{z}) = P_D^\mu(z)|_{z=\tilde{z}} + P_W^\mu(\tilde{w}), \quad (14)$$

respectively. Then, a non-oscillatory response is guaranteed by selecting either *series damping* (13) or *parallel damping* (14) satisfying the criteria of Theorem 1 or Theorem 2, respectively. It is interesting to remark that the concept of parallel damping as it follows from the modified Brayton and Moser criteria naturally coincides with the ideas as recently proposed in [2].

4 Tuning Example: The Boost Converter

In this section we will consider an illustrative example using the single-switch DC-to-DC Boost converter for the application of the modified Brayton-Moser criteria. One motivating reason for studying this circuit is that it describes in form and function a major family of power electronic converters. Consider the dynamic equations (5), but now with discrete states

²Here $\mathcal{R}(\cdot)$ and $\mathcal{G}(\cdot)$ are defined for the error dynamics. In comparison to the criterion in [1] this means that we have to take the opposite of $\mathcal{G}(z) + |\psi^T z| \rightarrow \infty$ for some $|z| \rightarrow \infty$, in the sense that $|z - \xi| \rightarrow 0$ has to hold in order to obtain a stabilizing controller that fulfills the requirements. The notation $\|K\|$ denotes the norm of K , defined as $\|K\|^2 = \max_{|x|=1} \{[Kx]^T Kx\}$.

x replaced by the average states z . Furthermore, for sake of simplicity we assume $R = 0$ and that the elements are all linear, time-invariant and perfectly known.

4.1 Series Damping PBC

In [5, 9] a passivity-based controller is proposed which, using the terminology proposed in the previous section, makes use of the series damping concept. For sake of brevity and lack of space we do not repeat the design procedure but only discuss the tuning problem using the resulting closed-loop error dynamics (for details see, e.g., [5]). For the series damping injection PBC the desired closed-loop co-energy and dissipation are set to $H_M^*(\bar{z}) = \frac{1}{2}L\bar{z}_1^2 + \frac{1}{2}C\bar{z}_2^2$ and $P_M^*(\bar{z}) = \frac{1}{2}G\bar{z}_2^2 + \frac{1}{2}R_i\bar{z}_1^2$, respectively, which results in the following controller

$$\mu = 1 - \frac{1}{\xi_2}[E - R_i(z_1 - \bar{\xi}_1)]$$

$$C\dot{\xi}_2 = \frac{\bar{\xi}_1}{\xi_2}[E - R_i(z_1 - \bar{\xi}_1)] - G\xi_2,$$

for every $\xi_2(0) > 0$, together with the closed-loop error-dynamics

$$L\dot{\bar{z}}_1 = -R_i\bar{z}_1 - (1 - \mu)\bar{z}_2$$

$$C\dot{\bar{z}}_2 = (1 - \mu)\bar{z}_1 - G\bar{z}_2, \quad (15)$$

where $R_i \geq 0$ is the injected series damping resistor and $\bar{\xi}_1$ denotes the desired value for the inductor current. It is easy to show using Lyapunov's stability theory that the closed-loop error system (15) is exponentially stable for every $G, R_i > 0$. However, we are interested to find a lower bound on R_i as to ensure a fast and non-oscillatory response. For the boost converter $\psi(\mu) = 1 - \mu$. Clearly, $\frac{1}{2}G\bar{z}_2^2 + |(1 - \mu)\bar{z}_2| \rightarrow 0$ as $|\bar{z}_2| \rightarrow 0$. Computing the norm of μ , we find $\|\psi(\mu)\|^2 \leq \mu$, for $\mu \in [0, 1]$, and therefore from Theorem 1 we have

$$\|K_R\|^2 = (1 - \mu) \frac{L}{R_i^2 C}.$$

Hence, a lower bound on R_i for a non-oscillatory response is obtained if and only if

$$R_i(\mu) > \sqrt{\frac{(1 - \mu)L}{C}}$$

for all $0 < \mu < 1$. Unfortunately, as is shown in [5], the series damping scheme is highly sensitive to unmodeled changes of the load conductance G . A possible solution is to extend the controller with an adaptive mechanism to compensate for such uncertainties [5]. The major disadvantage is that the resulting controllers become quite involved, even for simple systems like the boost converter. Another problem that arises is how to tune the adaptive controllers as to ensure stability and non-oscillatory responses. As is pointed out in [2], a simple solution to this problem is to use the concept of parallel damping injection to be treated next.

4.2 Parallel Damping PBC

The main idea behind the parallel damping injection scheme is the following. Let the desired closed-loop dissipation be

$$P_M^*(\bar{z}) = \frac{1}{2}(G + G_i)\bar{z}_2^2,$$

and take the closed-loop co-energy as before. For this case we find the controller as

$$\mu = 1 - \frac{E}{\xi_2}$$

$$C\dot{\xi}_2 = G\frac{\bar{\xi}_2^2}{\xi_2} - G\xi_2 + G_i(z_2 - \xi_2),$$

for every $\xi_2(0) > 0$, while $G_i \geq 0$ denotes the injected parallel damping conductance and $\bar{\xi}_2$ is the desired capacitor voltage. The corresponding closed-loop error-dynamics become

$$L\dot{\bar{z}}_1 = -(1 - \mu)\bar{z}_2$$

$$C\dot{\bar{z}}_2 = (1 - \mu)\bar{z}_1 - (G + G_i)\bar{z}_2, \quad (16)$$

One may interpret (16) as if there is an extra conductance connected in parallel to the output capacitor. Hence, for this case we have that $|(1 - \mu)\bar{z}_1| \rightarrow 0$ if $|\bar{z}_1| \rightarrow 0$, and thus the tuning criterion for G_i stemming from Theorem 2 is given by

$$G_i(\mu) > \sqrt{\frac{(1 - \mu)C}{L}} - G \quad (17)$$

for all $0 < \mu < 1$. Notice that the right-hand side of (17) may become negative. In that case the injected damping becomes negative as well, i.e., $G_i(\mu) < 0$. Strictly speaking, the controller then provides energy to the circuit and loses its passivity properties. On the other hand, consider the time-derivative of $H_M^*(\bar{z})$ along the trajectories of (16)

$$\dot{H}_M^*(\bar{z}) = -(G + G_i)\bar{z}_2^2 < 0, \quad \forall \bar{z} \neq 0.$$

It is easily checked from (17) that the closed-loop dissipative energy $P_M^*(\bar{z}_2)$ remains positive definite for all $G + G_i > 0$, even if $G_i < 0$. Hence, by using Lyapunov theory and by noting that (16) satisfies the Lipschitz condition, one can easily prove that the proposed controller indeed stabilizes the closed-loop dynamics of the system. Moreover, we conclude that the closed-loop remains passive for all $|G_p^i| < G_o < \infty$.

4.3 Simulation Experiments

Let us next test both the criteria using SIMULINK. We will use a Boost converter with the discrete values for the switch. This means that for the series damping injection scheme the only signal used for feedback is the 'real' inductor current $x_1 \geq 0$, and for the parallel damping scheme we only use 'real' capacitor voltage $x_2 \geq 0$. The design parameters of the Boost converter are chosen as follows: $E = 10V$, $L = 10\mu H$, $C = 50\mu H$, $R = G^{-1} = 5\Omega$ and the PWM switching frequency is set to $F_s = 50kHz$. The initial conditions are set to $x_1(0) = x_2(0) = 0$ and $\xi_2(0) = 1$. From Figure 1 we observe that for both schemes the controller rapidly stabilizes the capacitor voltages without any overshoot and oscillations. However, the series damping (top) does not reach the desired voltages \bar{x}_2 (dashed line), while the parallel scheme (center) reaches the setpoints within 2% accuracy. From this we may conclude that the series damping scheme is highly sensitive to the current ripples. This is due to the fact that the ripple in the inductor current is usually much higher than the ripple

in the output voltage. A better accuracy can be obtained by increasing the PWM frequency or by taking a larger inductor. Notice that the undershoot in the capacitor voltage is caused by the non-minimum phase nature of the converter.

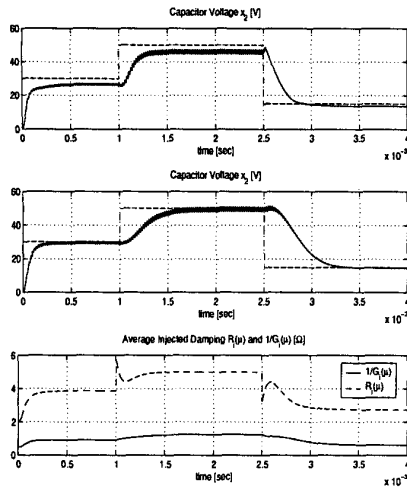


Figure 1: Closed-loop response for different setpoints \bar{x}_2 : (top) Series damping PBC; (center) Parallel damping PBC; (bottom) Average injected damping $R_i(\mu)$ and $G_i(\mu)$.

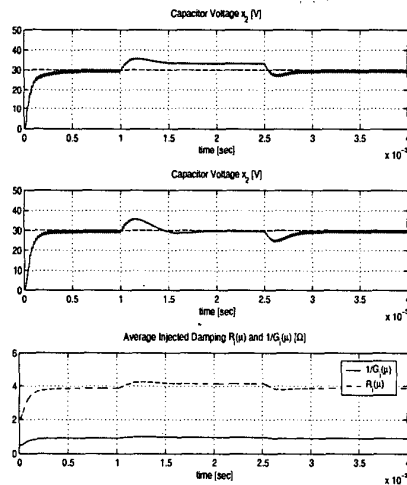


Figure 2: Closed-loop response for load perturbations ΔG : (top) Series damping PBC; (center) Parallel damping PBC; (bottom) Average injected damping $R_i(\mu)$ and $G_i(\mu)$.

Furthermore, Figure 2 shows the closed-loop response for load perturbations. These perturbations are set to $\pm 3G^{-1}\Omega$, while both schemes are adjusted to a nominal capacitor voltage of 30V. As can be seen, the parallel damping scheme rapidly manages to restore the capacitor voltage to its nominal value, while the series damping scheme does not manage to restore but forces the closed-loop to deviate from the desired voltage.

5 Conclusions

In this paper, the passivity-based design procedure for Euler-Lagrange systems is rewritten in terms of the Brayton-Moser equations. The advantage is that in this setting the states to be used for feedback are directly in terms of physically measurable quantities, i.e., currents and voltages. This in contrast to Lagrangian or Hamiltonian systems, whereas the coordinates are usually the charges and the fluxes, which in most cases can not be measured directly. Additionally, the assignment of parallel damping does in general not involve the use of current sensors but only needs the measurements of the voltages. A major advantage of parallel damping in comparison with series damping injection is that it does not require adaptive extensions in case the load resistors are unknown. Additionally, the idea of parallel damping injection provides a method to control non-minimum phase circuits based on the corresponding non-minimum phase output(s) only.

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